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A study on the teaching of Chinese Fangcheng method

By

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Abstract

The *Nine Chapters on the Art of Mathematics* has played a very important role and been considered as a classic in the history of Chinese mathematics. We formed a research team which used this material to introduce to high school students the setting of an equation, explaining its problem and method using the Fangcheng chapter of the Chinese classic. The main goal of this lesson plan was to introduce the Fangcheng method of solving systems of linear equations, so that students can naturally develop understanding of the concept of the matrix and its operations, and appreciate the Chinese cultural heritage as well. Thus, helping them realize that mathematics came from real life, we aroused students' interests. Among 33 respondents of the survey, 29 thought this course helped them understand the methods and basic concepts, and 28 said it enhanced their mathematical literacy. Obviously, introducing the Fangcheng chapter and Fangcheng method from the Chinese classic, and linking them to the learning of the matrix, we can enhance students' understanding and their beliefs about the cultural aspects of mathematics.

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§1. Preface

In Western civilization, mathematics has always been a major cultural force, and it can rival any kind of culture when it comes to providing pleasure and aesthetic value (see Klein [6]). The mathematics curriculum guidelines of the United States over the years have always attached great importance to the humanistic aspect of mathematical knowledge. For example, as early as 1989, the National Council of Teachers of Mathematics (NCTM) pointed out that, in order to understand the value of mathematics, students must be made aware of the interaction between mathematics and historical situations, and its impact on culture and life (see NCTM [3]). Again, in 2000, the aforementioned council stated that mathematics was one of the greatest cultural and intellectual achievements of mankind in this fast-changing world. Citizens should develop appreciation and understanding of this achievement, including its beauty and even its entertaining aspect (see NCTM [4]). Fasanelli [1] analyzed the curricula of China, Greece, Italy, the Netherlands, Poland, Australia, Brazil, Denmark, France, New Zealand, Norway and other countries and discovered that the history of mathematics can inspire the affective, cognitive and cultural development of a person.

Taiwan's Ministry of Education stressed the following in its nine-year National Curriculum for Primary and Secondary Public Schools: "Introducing the history of mathematics in teacher-training programs have positive effects for students' learning. In particular, [the history of mathematics] helps students form concrete conceptions for abstract mathematical ideas." (Refer to Ministry of Education [11]). The guideline of the recently promulgated 12-year national Basic Education Mathematics Curriculum also mentions the humanistic aspect of mathematics: "The diversities among the development of different human civilizations and societies have created distinct thinking patterns and cultures. For example, ancient Oriental mathematics was prone to concrete inductive reasoning, whereas the Occidental counterpart was prone to abstract deductive thinking. However, these two approaches are merely presenting in different languages the realities behind complex phenomena. Therefore, the history of mathematics can help us understand the differences of mathematical developments in different cultures." One can clearly see how using materials from the history of mathematics in teaching can have a positive supporting role.

Whether in elementary, middle school or high school, we have discovered that when integrating the history of mathematics in the teaching, the students were not only able to understand better the hidden mystery of mathematics, but were also equipped with a positive learning attitude (see Shen Chih-Lung et al. [7], Lin Miao-Shuang, et al. [8] and Tsai Hsing-Ni et al. [12]). In fact, integrating humanistic materials in teaching mathematics in high schools, middle schools and even elementary schools can help students recognize that mathematics is a very human activity. We help students realize that the development of mathematical thinking is closely linked with the social and cultural contexts (see Horng Wann-Sheng [9], Horng Wann-Sheng et al. [10]). From the above discussions, we can see how important materials from the history of mathematics are in mathematics teaching. However, it is not easy to find relevant and well-constructed materials in the history of mathematics suitable for teaching. Even if teachers are determined to implement it, the first problem they face is the question of where to look for adequate materials (see Su Yi-Wen [14]).

An online survey of 367 secondary school mathematics teachers by Panasuk and Horton [5] also showed that while most teachers agree with the value and status of the history of mathematics in teaching, the lack of related resources is one of the main reasons for avoiding using the history of mathematics in class. Therefore, in this study, teacher educators and field teachers jointly planned to develop teaching materials of high school mathematics from the history of mathematics, hoping to facilitate its use among teachers in the field. We hope that through this approach, students can develop mathematical thinking and better understanding of mathematical concepts, and finally enhance their interest in learning mathematics.

In this context, we formed a research team for high school mathematics, and we studied and discussed related history of mathematics literature in the following areas: connotation of limits, connotation of derivatives, methods of finding extreme values, segments and integrals, expectations and binomial expansions, the names of trigonometric functions, linear equations and matrixes, and Ptolemy's theorem and trigonometric identities. The development of materials on the history of mathematics and related teaching plans were subjected to three phases by experts as follows: validity check, field trial, and evaluation of its effectiveness. This paper explores the teaching practices and effectiveness of the lesson plans on linear equations and matrixes.

§2. The context in Chinese Mathematics used in the Teaching Practices

Chinese mathematics, as many scholars believe, is a system in which practitioners start from realistic problems and goes through processes of analysis, to find general principles and methods, and eventually solve the original problems. Chinese mathematicians constructed computational models for realistic problems, and found solutions with mechanical algorithms. Mechanical algorithm is one of the most distinctive characteristics of Chinese mathematics, as some scholars claim. This is why mathematics in Chinese is called *suanxue* [算學], i.e., “the study of calculations.” (Liu Dun [13]).

The *Nine Chapters on the Art of Mathematics* [九章算術] (abbreviated *Nine Chapters* in the sequel) was compiled no later than first century AD, in the later Han Dynasty, in which 246 problems and solutions were collected and put into nine categories. The *Nine Chapters* occupies a very important role in the history of Chinese mathematics, as Rong Qi [榮棨] wrote in the preface of its publication in the 12th century, as follows: "The *Nine Chapters* is at the head of mathematical canons, just as the *Six Classics* is for Confucianism, the *Nanjing* [難經] and the *Suwen* [素問] for medicine, and the *Sun Zi* [孫子] for the art of war. If any later scholar could have a glimpse of the contents or follow a fraction of the methods, maybe he could become a master of a school and be known to the world."

As a result, we took the methods introduced in the 8th chapter of the *Nine Chapters* in our teaching experiment for the matrix and solving systems of linear equations. The lesson plan first presents the meaning of the title of the 8th chapter, Fangcheng [方程]:

> 程，課程也。群物總雜，各列有數，總言其實。今每行為率，二物者再程，三物者三程皆如物數程之，并列為行，故謂之方程。
> The *`cheng'* [程] is the same as *`ke cheng'* [課程], which means comparing measures. Groups of things are mingled together, each is given a number, and in every case their total is stated. Let each column represent the different rates. If there are two things, there are two rates of measures; if three things, three rates of measures. The number of rates of measures depends on the number of things. Altogether they are arranged in columns, and that is why it is called Fangcheng, or "juxtaposing measures" ("Rectangular Arrays")."

As the reader can see, the term Fangcheng cannot simply be translated as "equation". Originally the term cheng [程] means "the measure of things", while fang [方] means "to put together". So Fangcheng is to put several measures of things together, in order to investigate them. Our translation and interpretations of the method and related problems of Fangcheng relied mainly on Guo, Dauben and Xu [2].

The research team used the Fangcheng method to introduce the equation setting and unknown elimination by explaining the first problem and method in the 8th chapter in the *Nine Chapters*. The first problem in the chapter reads as follows:

> 今有上禾三秉，中禾二秉，下禾一秉，實三十九斗；上禾二秉，中禾三秉，下禾一秉，實三十四斗；上禾一秉，中禾二秉，下禾三秉，實二十六斗。問上、中、下禾，實一秉幾何？
> Suppose the number of bundles of top-grade millets is 3; the number of bundles of medium-grade millets is 2; the number of low-grade millets is 1; the total capacity is 39 *dou* (bushels). The number of bundles of top-grade millets is 2; the number of bundles of medium-grade millets is 3; the number of low-grade millets is 1; the total capacity is 34 *dou*. The number of bundles of top-grade millets is 1; the number of bundles of

medium-grade millets is 2; the number of low-grade millets is 3; the total capacity is 26 *dou*. Please find the capacity of 1 bundle of each kind of millets.

This problem, if translated into modern symbolism, is of course a standard problem for solving a system of linear equations. The research team wished to use this material from the history to introduce to high school students the setting of an equation, explaining its problem and method by means of the Fangcheng chapter of the *Nine Chapters*.

§3. Actual implementation of the teaching

The main goal of this lesson plan was to introduce the Fangcheng method of solving linear equations in detail, so that students can naturally develop their understanding of the concept of the matrix and its operations, and appreciate as well the Chinese cultural heritage from the *Nine Chapters*. In turn, we might help them realize that mathematics came from real world and arouse their interest.

In the present-day high school mathematics textbooks in Taiwan, a lesson is usually arranged as follows: starting from definitions and theorems and then going to formulas and problems. In the *Nine Chapters*, it starts with the problems and then gives the procedures (formulas) without telling the reasons.

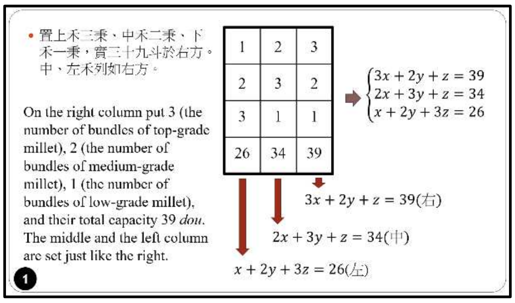
From the differences of the formulas we could introduce the cultural perspective. We let students start from real problems to set up linear equations, and then they tried to eliminate unknowns and find solutions. The lesson was conducted in 45 minutes to a class of 34 students who aimed to study humanities or social sciences in university. They already learned and had examination about the topics of the matrix and row operations. The teacher used lecturing, learning sheets, teacher-student interactions with Q&A as his teaching methods, and the contents were taught with the assistance of slides.

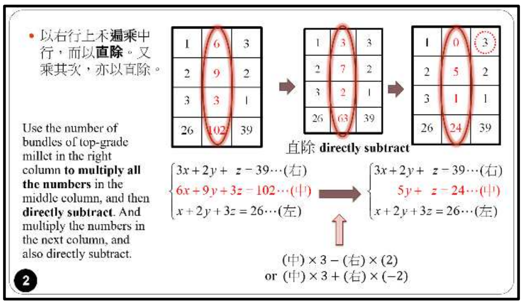
At first, the teacher distributed the first page of the work sheets, indicating that the problem of linear equations has a long history and that can be found in practical needs. The problem of the top-grade, medium-grade and lower-grade millet in the same volume having different weights as shown in the illustration is presented to them. Then, the teacher introduced ancient Chinese mathematics in solving such systems of equations by introducing the *Nine Chapters* as a classic and representative example. Second, the teacher distributed the second page of work sheet, showing how in the *Nine Chapters* this type of equations is found in its Fangcheng chapter. The students were then asked to read the Fangcheng chapter and write the relevant equations to the problem. In 31 minutes, all students correctly wrote down 3 equations as follows. $3x+2y+z=39$, $2x+3y+z=34$, $x+2y+3z=26$. The second question asked the students the values of the unknowns: $x=\dfrac{37}{4}$, $y=\dfrac{17}{4}$, $z=\dfrac{11}{4}$.

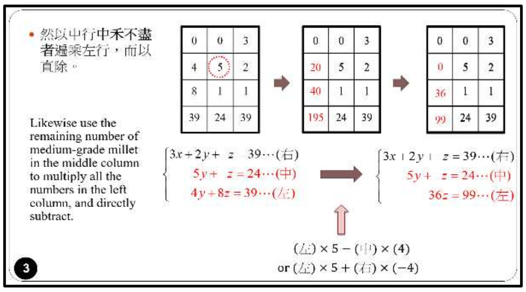
Students' solutions varied. 20 students used the method of elimination by addition and subtraction (or "unknown elimination method") to solve the problem; 7 students used matrix operations; 4 students did not answer the question.

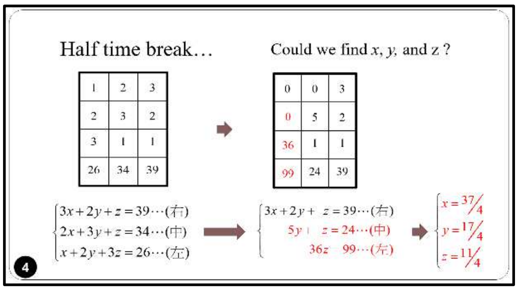
Next the teacher asked the students "What do you think is the possible weakness of this 'unknown elimination method'?" A total of 18 students answered the question, which can be summed up into two points: (1) when the numbers are too large and with many variables, using "unknown elimination method" becomes too complex and the formulas are messy; (2) if the answer is somewhat "ugly", then it might also be hard to calculate.

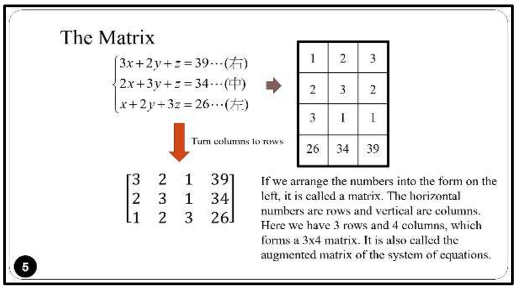
After the discussion, the teacher introduced the paragraph for the procedure of the Fangcheng method, leaving a few blanks for students to fill. 30 of the 34 students answered correctly. Later on, the teacher used slides to explain Fangchang formula of equation using clear common language, presenting how to find the correct answer as shown in the following illustrations:

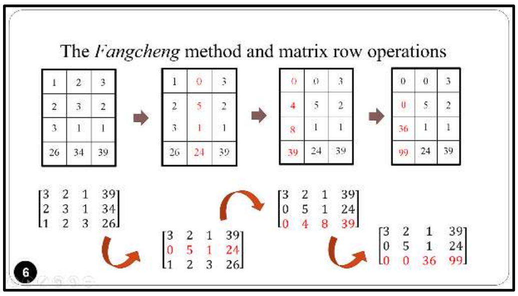












The teachers asked the students to point out the two basic operations in this procedure for Fangcheng method, that is, "to multiply all the numbers" (*biancheng* 遍乘) and "to directly subtract" (*zhichu* 直除). Twenty-two students got the correct answer. Finally, the students were asked to compare the Fangchang method with the unknown elimination method of solving such problems. Only 17 students answered, perhaps because there was not sufficient time. There was consistency in the way the students solved the problems, in which they generally considered Fangchang method (1) did not need pencil calculation and the numbers were aligned without confusion, and (2) was a standardized procedure so there is little chance of mistakes.

The teachers realized that after the implementation of the course, the schedule was very tight that even with the help of slides, students were barely able to complete the entire work sheet in 45 minutes, which was not ideal for students to answer well. However, from the feedback sheets, we can conclude that the teaching generally reached the expected learning result.

§4. Concluding remarks and reflections

After the class, the teacher asked the students to write their suggestions, thoughts and reflections regarding the teaching and content of the topic on linear equations and matrixes. The following are some of their answers:

2 students wrote that the teacher could have talked more about the weaknesses of Fangcheng method. It would have been interesting to compare it with the matrix.

3 students believed that ancient mathematics and names were interesting, such as the top-, medium-, and low-grade millets. The ancient problem interested the students, and they understood that mathematics was not just mathematics, but came from daily lives.

1 student wrote: Learning mathematics for modern people is luckier than for ancient people. Their method was so messy! It takes a lot of patience. It's good to be a modern person. (Arabic numbers are easier to use.)

11 students thought that it was fun and interesting. They learned some things they never heard before. Ancient Chinese were so smart.

8 students replied that they had always studied and written mathematics using Western perspectives. On that day they saw the faces of ancient Chinese mathematics and felt interested.

Furthermore, among the 33 respondents of the survey, 29 thought this course helped them understand the methods and basic concepts, and 28 said it enhanced their mathematical literacy. Obviously, introducing the Fangcheng chapter and its method from the *Nine Chapters*, and linking them to the learning of matrix, we could enhance students' understanding about the cultural aspects of mathematics. Besides, students could also feel the connections between mathematics and real life.

A few students did not have very positive responses to Fangcheng method, possibly because they were not used to the operations of "to multiply all the numbers" and "to directly subtract". One student even said the method is messy. The research team believe that these two operations are indeed more suitable for counting rod calculations instead of pencil calculations. However, the current teaching plan is still valuable because it helps students build up concepts for solving a system of linear equations, and it shows students the cultural value of mathematic. The research team further reflected that the process also extended the necessary teaching time, and allowed students more time to acquire a deeper thinking and considerations about the topic. Only with the extended time could the students be able to fully absorb the lesson. We hope that in the future we can try to integrate more material of ancient mathematics into modern classes.

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